MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

21 OCTOBER 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the formulae sheet and statistical table.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [25 marks]

(a) The random variable X has the CDF of:

$$F(x)=\begin{cases}0 & , \quad x<1\\ 1/2 & , \quad 1\le x<2\\ 3/4 & , \quad 2\le x<3\\ 7/8 & , \quad 3\le x<4\\ 1 & , \quad x\ge 4\end{cases}$$

(i) Construct the probability distribution for X. [2 marks]
(ii) Determine the mean and variance for X. [8 marks]
(iii) Determine the MGF for X. [3 marks]
(iv) Verify E(X) in part (ii) using the MGF of X. [5 marks]

(b) A study of the time spent shopping in a supermarket for a market basket of 20 specific items showed an approximately uniform distribution between 15 minutes and half an hour.
(i) What is the probability that the shopping time will be less than 25 minutes? [3 marks]
(ii) What is the probability that the shopping time will be between 20 minutes and 50 minutes? [2 marks]
(iii) Determine the mean time spent shopping in a supermarket for a market basket of 20 specific items. [2 marks]

Question 2 [25 marks]

(a) The random variable X and Y has joint pdf as given

$$f(x,y)=k(x+2y) \quad ; \quad 0\le x\le 1 \quad ; \quad 0\le y\le 1$$

(i) Determine the value of k. [6 marks]
(ii) Determine the marginal pdf for X and Y respectively. [8 marks]
(iii) Are X and Y independent? Explain. [5 marks]

(b) Suppose that past history shows that 60% of college students are smokers. A sample of 5 students is to be selected.
(i) Find the probability that at most 3 students are smoker. [3 marks]
(ii) Find the probability that 2 or 3 students are non-smoker. [3 marks]

Continued...

Question 3 [25 marks]

(a) Customers arrive at the neighborhood bookstore with an average rate of 10 customers per hour. The bookstore is currently doing a promotion for giving out RM5 cash voucher to the first 3 customers entering the bookstore since 10am.

(i) Find the probability for the second cash voucher to be distributed within 25 minutes. [7 marks]

(ii) Find the probability to finish distributing all the cash vouchers of the day before 11am. [9 marks]

(b) Ben and Jerry play a series of games of checkers. Each game cost \$1 from each player. Ben is a better player than Jerry and has a 60% chance of winning a game. Initially Ben had \$9, while Jerry has triple of Ben's initial capital.

(i) Find the probability that Ben is ruined. [5 marks]

(ii) Find the expected duration for the game. [4 marks]

Question 4 [25 marks]

Consider a Markov chain with state {1, 2, 3}and transition probability matrix

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Determine for every state whether it is an absorbing state. [3 marks]

(b) Draw the transition diagram. [3 marks]

(c) Determine whether state 1 and state 2 to be transient or persistent state. [9 marks]

(d) Determine the period for state 1 and 2. [4 marks]

(e) Find $P_{13}^{(2)}$. [6 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution $P(X = x) = p^x q^{1-x}$	for $x = 0, 1$
Binomial Probability Distribution $P(X = x) = {}^nC_x\, p^x q^{n-x}$	for $x = 0, 1, \ldots, n$
Poisson Probability Distribution $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$	for $x = 0, 1, 2, \ldots$
Geometric Probability Distribution $P(X = x) = pq^{x-1}$	for $x = 1, 2, \ldots$
Uniform Probability Distribution $f(x) = \frac{1}{b-a}$	for $a < x < b$
Exponential Probability Distribution $f(x) = \lambda e^{-\lambda x}$	for $x > 0$
Normal Probability Distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	for $-\infty < x < \infty$

B. MOMENT

k^{th} Moment about the origin of X: $\mu'_k = E[X^k]$
k^{th} Moment about the mean of X: $\mu_k = E[X - \mu]^k$
Moment Generating Function for X: $M_X(t) = E\left(e^{tx}\right)$
Derivative of Moment Generating Function for X: $M_X^{(r)}(t) = E\left(X^r e^{tx}\right)$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where m and n are both even or both odd.

$E(X_n) = n(p - q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{(\frac{q}{p})^a - (\frac{q}{p})^c}{1-(\frac{q}{p})^c}, & p \neq q \\ 1-\dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = a(c-a) \quad \text{if} \quad p = q$$

$$D_a = \frac{1}{q-p}\left\{a - c\left[\frac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} \quad \text{if} \quad p \neq q$$

D. POISSON PROCESS

Pmf for N(t) :

$$P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

Pdf for T_n:

$$f(t) = \lambda e^{-\lambda t}$$

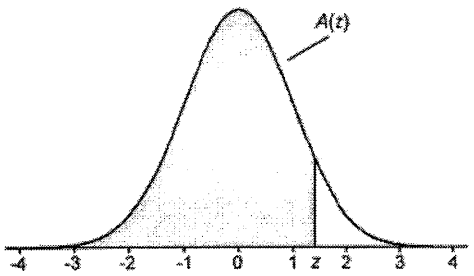
Pdf for S_n :

$$f(t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$

STATISTICAL TABLES 1

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							